A NEW FORMALISM OF INCLUSIVE BREAKUP REACTIONS AND VALIDITY OF THE SURFACE APPROXIMATION ☆

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Received 21 April 1982
Revised manuscript received 27 May 1982

Austern and Vincent's formalism for the inclusive breakup reactions in DWBA is rewritten in a form in which the elastic and inelastic breakup processes are well discernible. This form is calculable exactly and hence we test the validity of the surface approximation used by Baur et al.

Recently inclusive breakup processes have attracted much interest in light- as well as heavy-ion induced reactions. Baur and collaborators [1,2] decomposed the process, a + A → b + anything with a = b + x, into two parts; the elastic breakup, a + A → b + x_{gr} + A_{gr} (gr denoting the ground state) and the rest which is called the inelastic breakup. They demonstrated that the latter contributed the dominant part of the total inclusive breakup cross sections for the A(d, p), A(α, ^{3}He) reactions. Udagawa and collaborators [3] also showed that the inelastic breakup part dominates heavy-ion induced inclusive breakup reactions such as ^{40}Ca(^{20}Ne, ^{16}O), ^{159}Tb(^{14}N, α), etc. They called the inelastic breakup breakup-fusion.

The method of calculation of Baur et al. [1,2] is based on the post-form DWBA using the surface approximation (SA) for the form factor and the unitarity of the *S*-matrix for the x–A collision. This method was first proposed by Vincent and Fortune [4]. Udagawa and Tamura [5], on the other hand, used the prior-form DWBA and the on-energy-shell approximation for the progagator of the x–A system (hereafter called the system B), starting from Kerman and McVoy's two-step formalism [6] and their different derivation [7]. Very recently Udagawa et al. [8] reported an improved calculation of the breakup- fusion for the ^{181}Ta(^{14}N, α) reaction without using the on-energy-shell approximation.

Recently Austern and Vincent [9] derived a closed formula for inclusive breakup cross sections with an optical-model Green's function of the system B, starting with the post-form DWBA. They also proposed some exact and approximate forms which are suitable for numerical calculations, and discussed the relation with the SA of Baur et al. [1,2].

In this short note, we first show a simple improvement of the elastic–inelastic breakup decomposition of Austern–Vincent's formula. We secondly show that the expression derived is easily calculable without approximations. The expression turns out to be very similar to that of Udagawa et al. [8]. We also demonstrate that its connection with the SA is very transparent. Then, numerical calculations by this formula are carried out for the inclusive cross sections of the inelastic deuteron breakup process and the validity of the SA is investigated.

☆ Supported in part by NSF and DOE under Contract No. PHY-8109019 and DE-AS05-80ER10565, respectively.

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For simplicity and comparison with works of Austern and Vincent [9] and Baur et al. [1,2] we consider, as an example, the deuteron breakup process

$$d + A \to p + B_c ,$$

where B_c specifies the final channel of the system B, and to identify the $n + A_{gr}$ channel B_0 will be used. We follow the terminology of Baur et al., "inelastic breakup-fusion", instead of "breakup-fusion" since we compare our results with theirs.

In the post-form DWBA for the proton emission, the inclusive breakup cross section is expressed by Austern and Vincent [9] as

$$d\sigma^{\mathrm{incl}} = \frac{(2\pi)^4}{\hbar v_d} \sum_c |T^{\mathrm{DWBA}}_{d,p_c}|^2 \delta(E - E_p - E_{B_c}) d^3k_p$$

$$= \frac{-(2\pi)^4}{\pi\hbar v_d} \mathrm{Im} \sum_c \frac{|\langle \chi^{(-)}_{k_p} \Phi^{(-)}_{B_c} | V_{pn} | \chi^{(+)}_{k_d} \phi_d \Phi_A \rangle|^2}{E^\dagger - E_p - E_{B_c}} d^3k_p$$

$$= \frac{-(2\pi)^4}{\pi\hbar v_d} \mathrm{Im} \{ \langle \chi^{(+)}_{k_d} \phi_d | V_{pn} | \chi^{(-)}_{k_p} \rangle$$

$$\times \langle \Phi_A | (E^\dagger - E_p - H_B)^{-1} | \Phi_A \rangle \langle \chi^{(-)}_{k_p} | V_{pn} | \chi^{(+)}_{k_d} \phi_d \rangle \} , \quad (1)$$

where $H_B \Phi^{(-)}_{B_c} = E_{B_c} \Phi^{(-)}_{B_c}$, and $\chi^{(+)}_{k_\alpha}(\alpha = p, d)$ are the distorted waves of the particle α specified by the momentum k_α, and Φ_A and Φ_d are the intrinsic ground state wave functions of the nucleus A and the deuteron, respectively. Averaging eq. (1) over energy, one obtains the energy-averaged cross section as

$$d\bar{\sigma}^{\mathrm{incl}} = -[(2\pi)^4/\pi\hbar v_d] \, \mathrm{Im} \langle \rho | G_n(E^\dagger_n) | \rho \rangle \quad (2)$$

$$= -[(2\pi)^4/\pi\hbar v_d] \langle \rho | \mathrm{Im} \, G_n(E^\dagger_n) | \rho \rangle , \quad (2')$$

where $E_n = E - E_p - E_{A_{gr}}$ and

$$\rho = \langle \chi^{(-)}_{k_p} | V_{np} | \chi^{(+)}_{k_d} \phi_d \rangle \quad (3)$$

is the neutron source function, and

$$G_n(E_n) = (E^\dagger_n - k_n - U_n)^{-1} \quad (4)$$

is the optical-model Green's function of the neutron in the optical potential U_n from the nucleus A. To obtain eq. (2'), the symmetry of G_n is used. The expression (2) is the closed form derived by Austern and Vincent [9]. From the equations

$$G_n = G_{0n} + G_{0n} U_n G_n , \quad (5)$$

$$G_{0n} \equiv (E^\dagger_n - k_n)^{-1} , \quad \mathrm{Im} \, G_{0n} = -\pi\delta(E_n - k_n) , \quad (6)$$

one can derive the identity

$$\mathrm{Im} \, G_n = (1 + G^\dagger_n U^\dagger_n)[-\pi\delta(E_n - k_n)] (1 + U_n G_n)$$

$$+ G^\dagger_n (\mathrm{Im} \, U_n) G_n . \quad (7)$$

Inserting eq. (7) into eq. (2'), one obtains

$$\frac{d\bar{\sigma}^{\mathrm{incl}}}{d^3k_p} = \frac{(2\pi)^4}{\hbar v_d} \langle \rho | (1 + G^\dagger_n U^\dagger_n) \delta(E_n - k_n)(1 + U_n G_n) | \rho \rangle$$

$$- \frac{(2\pi)^4}{\pi\hbar v_d} \langle \rho | G^\dagger_n (\mathrm{Im} \, U_n) G_n | \rho \rangle . \quad (8)$$

The neutron distorted wave is introduced as

$$\chi^{(-)}_{k_n} = (1 + G^\dagger_n U^\dagger_n) | k_n \rangle , \quad (9)$$

where $K_n | k_n \rangle = (\hbar^2 k_n^2 / 2\mu_n) | k_n \rangle$, then, the first term of eq. (8) is rewritten as

$$\frac{(2\pi)^4}{\hbar v_d} \int d^3k_n |\langle \rho | \chi^{(-)}_{k_n} \rangle|^2 \delta(E_n - \hbar^2 k_n^2 / 2\mu_n)$$

$$= \frac{(2\pi)^4}{\hbar v_d} \int d^3k_n |\langle \chi^{(-)}_{k_p} \chi^{(-)}_{k_n} | V_{np} | \chi^{(+)}_{k_d} \phi_d \rangle|^2$$

$$\times \delta\left(E_n - \frac{\hbar^2 k_n^2}{2\mu_n}\right) . \quad (10)$$

Hence one finds that the first term of the rhs of eq. (8) is nothing but the energy-averaged elastic breakup cross section, $d\bar{\sigma}_{\mathrm{el}}/d^3k_p$.

The inclusive cross section is thus decomposed into the elastic and inelastic parts as

$$d\bar{\sigma}^{\mathrm{incl}} = d\bar{\sigma}_{\mathrm{el}} + d\bar{\sigma}_{\mathrm{inel}} . \quad (11)$$

The energy averaged inelastic breakup cross section is then given by

$$\frac{d\bar{\sigma}_{\mathrm{inel}}}{d^3k_p} = \frac{(2\pi)^4}{\pi\hbar v_d} \int d^3r \, W_n(r) \left| \int G_n(r, r') \rho(r') \, d^3r' \right|^2 , \quad (12)$$

with $W_n(r) = -\mathrm{Im} \, U_n(r)$ by assuming that U_n is local.

The expression (12) is very similar to the formalism of Kerman and McVoy [6], and Udagawa et al. [7,8]. A difference is that in the present case, the interaction responsible for the breakup (V_{np}) does not excite the

target A, and therefore, only the ground state component of the n–A optical potential is needed without approximations.

For numerical calculations, Austern and Vincent [9] advocate an expression

$$\frac{\mathrm{d}\bar{\sigma}^{\mathrm{incl}}}{\mathrm{d}^3k_\mathrm{p}} = \frac{(2\pi)^4}{\hbar v_\mathrm{d}} \int \mathrm{d}^3k_\mathrm{n} |\langle \boldsymbol{k}_\mathrm{n}|\rho\rangle|^2 \delta(E_\mathrm{n} - \hbar^2 k_\mathrm{n}^2/2\mu_\mathrm{n}) + \frac{(2\pi)^4}{\pi\hbar v_\mathrm{d}} \mathrm{Im} \int \rho^*(r) G_{0\mathrm{n}}(r, r') U_\mathrm{n}(r') \times G_\mathrm{n}(r', r'') \rho(r'')\, \mathrm{d}^3r\, \mathrm{d}^3r'\, \mathrm{d}^3r'' , \quad (13)$$

by inserting eq. (5) in eq. (2). Due to the short range property of $U_\mathrm{n}(r)$, the second integration can be readily performed. However, the integral (12) is much simpler than this integral. It is also noted that the calculation of the elastic breakup is well established [10]. We thus recommend use of the expressions (11) and (12), instead of the expression (13).

In the zero-range approximation with the finite-range correction [10], the neutron source function is written as

$$\rho(r) = D_0 \chi^{(-)*}_{\boldsymbol{k}_\mathrm{p}}(cr)\, \chi^{(+)}_{\boldsymbol{k}_\mathrm{d}}(r) \Lambda(r) \quad (14)$$

where D_0 is the "zero-range constant" and $\Lambda(r)$ is the finite-range correction factor and $c = m_\mathrm{A}/(m_\mathrm{A} + m_\mathrm{n})$. It is expanded by spherical harmonics as

$$\rho(r) = \frac{D_0}{\pi^{3/2} k_\mathrm{p} k_\mathrm{d} c} \sum_{l_\mathrm{n} m_\mathrm{n}} \sum_{l_\mathrm{p} l_\mathrm{d}} \rho^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r, k_\mathrm{p}, k_\mathrm{d}) \times Y_{l_\mathrm{n} m_\mathrm{n}}(\hat{r})\, \mathcal{Y}^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n} m_\mathrm{n}}(\hat{\boldsymbol{k}}_\mathrm{p}, \hat{\boldsymbol{k}}_\mathrm{d}) , \quad (15)$$

with

$$\mathcal{Y}^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n} m_\mathrm{n}}(\hat{\boldsymbol{k}}_\mathrm{p}, \hat{\boldsymbol{k}}_\mathrm{d}) = \sum (l_\mathrm{p} m_\mathrm{p} l_\mathrm{d} m_\mathrm{d} | l_\mathrm{n} m_\mathrm{n})\, Y_{l_\mathrm{p} m_\mathrm{p}}(\hat{k}_\mathrm{p})\, Y_{l_\mathrm{d} m_\mathrm{d}}(\hat{k}_\mathrm{d}) . \quad (16)$$

The Green's function $G_\mathrm{n}(r, r')$ is also expanded in the form

$$G_\mathrm{n}(r, r') = -\frac{2\mu_\mathrm{n}}{\hbar^2 k_\mathrm{n}} \sum_l \frac{f_l(r_<) h_l(r_>)}{rr'} \sum_m Y_{lm}(\hat{r})\, Y^*_{lm}(\hat{r}') , \quad (17)$$

where f_l and h_l are regular and outgoing radial wave functions for the potential U_n, respectively. The integral

$$\langle r|G_\mathrm{n}\rho\rangle = \int G_\mathrm{n}(r, r')\, \rho(r')\, \mathrm{d}^3r'$$

in eq. (12) is then expressed as

$$\langle r|G_\mathrm{n}\rho\rangle = -\frac{2\mu_\mathrm{n} D_0}{\pi^{3/2}\hbar^2 k_\mathrm{p} k_\mathrm{d} k_\mathrm{n} c} \sum_{l_\mathrm{n} m_\mathrm{n}} r^{-1} Y_{l_\mathrm{n} m_\mathrm{n}}(\hat{r}) \times \sum_{l_\mathrm{d} l_\mathrm{p}} R^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r) \mathcal{Y}^{l_\mathrm{p} l_\mathrm{d}*}_{l_\mathrm{n} m_\mathrm{n}}(\hat{\boldsymbol{k}}_\mathrm{p}, \hat{\boldsymbol{k}}_\mathrm{d}) , \quad (18)$$

where

$$R^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r) = \int_0^\infty r' f_{l_\mathrm{n}}(r_<)\, h_{l_\mathrm{n}}(r_>) \rho^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r')\, \mathrm{d}r' . \quad (19)$$

Finally, the inelastic cross section (12) is given by

$$\left(\frac{\mathrm{d}^2\bar{\sigma}}{\mathrm{d}E_\mathrm{p}\mathrm{d}\Omega_\mathrm{p}}\right)_\mathrm{inel} = \frac{2^6 \mu_\mathrm{d}\mu_\mathrm{p}\mu_\mathrm{n}^2}{\hbar^8 k_\mathrm{d}^3 k_\mathrm{p} k_\mathrm{n}^2} \frac{D_0^2}{c^2} \sum_{l_\mathrm{n} m_\mathrm{n}} I_{l_\mathrm{n} m_\mathrm{n}}(\boldsymbol{k}_\mathrm{p}, \boldsymbol{k}_\mathrm{d}) , \quad (20)$$

where

$$I_{l_\mathrm{n} m_\mathrm{n}}(\boldsymbol{k}_\mathrm{p}, \boldsymbol{k}_\mathrm{d}) = \int_0^\infty \mathrm{d}r\, W_\mathrm{n}(r) \left| \sum_{l_\mathrm{p} l_\mathrm{d}} R^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r)\, \mathcal{Y}^{l_\mathrm{p} l_\mathrm{d}*}_{l_\mathrm{n} m_\mathrm{n}}(\hat{\boldsymbol{k}}_\mathrm{p}, \hat{\boldsymbol{k}}_\mathrm{d}) \right|^2 . \quad (21)$$

The angle-integrated proton energy spectrum is then given by

$$\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}E_\mathrm{p}}\right)_\mathrm{inel} = \sum_{l_\mathrm{d}} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}E_\mathrm{p}}\right)^{l_\mathrm{d}}_\mathrm{inel} = \sum_{l_\mathrm{d}} \sum_{l_\mathrm{p} l_\mathrm{n}} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}E_\mathrm{p}}\right)^{l_\mathrm{d} l_\mathrm{p} l_\mathrm{n}}_\mathrm{inel} , \quad (22)$$

with

$$\left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}E_\mathrm{p}}\right)^{l_\mathrm{d} l_\mathrm{p} l_\mathrm{n}}_\mathrm{inel} = \frac{2^6 \mu_\mathrm{d}\mu_\mathrm{p}\mu_\mathrm{n}^2 D_0^2}{\hbar^8 k_\mathrm{d}^3 k_\mathrm{p} k_\mathrm{n}^2 c^2} \times (2l_\mathrm{n} + 1) \int_0^\infty \mathrm{d}r\, W_\mathrm{n}(r) |R^{l_\mathrm{p} l_\mathrm{d}}_{l_\mathrm{n}}(r)|^2 . \quad (23)$$

Now let us discuss the SA of Baur et al. from our

formalism. Their assumption is that there exists a radius R_c such that

$$\rho_{l_n}^{l_p l_d}(r) \approx 0 , \quad \text{for } r < R_c ,$$

$$W_n(r) \approx 0 , \quad \text{for } r > R_c , \tag{24}$$

because of the Coulomb repulsion or the absorption of the proton and deuteron waves. Then the integral (21) turns out to be

$$I_{l_n m_n} \approx \left(\int_0^{R_c} W_n(r) |f_{l_n}(r)|^2 \, dr \right)$$

$$\times \left| \int_{R_c}^{\infty} h_{l_n}(r) \sum_{l_p l_d} \rho_{l_n}^{l_p l_d}(r) r \, dr \mathcal{Y}_{l_n m_n}^{l_p l_d}(\hat{k}_p, \hat{k}_d) \right|^2 . \tag{25}$$

Noting that the first factor is related to the partial reaction cross section $\sigma_{l_n}^{r}$ of the n–A collision as

$$\int_0^{R_c} W_n(r) |f_{l_n}(r)|^2 dr \approx \int_0^{\infty} W_n(r) |f_{l_n}(r)|^2 dr = \frac{\hbar^2 k_n}{2\mu_n} \sigma_{l_n}^{r} , \tag{26}$$

one notices that our formalism with the assumption (24) exactly leads to the SA of Baur et al. [1,2]. This derivation is more direct than that of Austern and Vincent [9] since they needed extra approximations. To avoid dependence on R_c, Bauer et al. [1,2] modified the second factor of eq. (25) by replacing $\int_{R_c}^{\infty} h_{l_n}(r)$ by $\int_0^{\infty} r[\chi_{l_n}(r) - j_{l_n}(r)]/(S_{l_n l_n} - 1)$ where $j_{l_n}(r)$ is the spherical Bessel function, $\chi_{l_n}(r)$ is the neutron distorted waves with angular momentum l_n and $S_{l_n l_n}$ is the S-matrix element of the elastic n + A scattering with l_n.

Our final expression (20) with eq. (21) is easily calculable without the approximations Bauer et al. and Austern et al. used. A difficulty in calculation is poor convergence of the integral (19) but it can be managed by the technique of Vincent and Fortune [11].

By the present formalism we calculate the inelastic breakup cross sections of ^{93}Nb(d, p) and ^{62}Ni(d, p) at E_d = 25.5 MeV, which Pampus et al. [2] analysed by the SA. For the sake of comparison we use the same optical parameters as they used. In fig. 1, the double differential cross section $(d^2\bar{\sigma}/d\Omega_p \, dE_p)_{incl}$ of ^{93}Nb(d, p) calculated by the SA [2] (dotted line) is

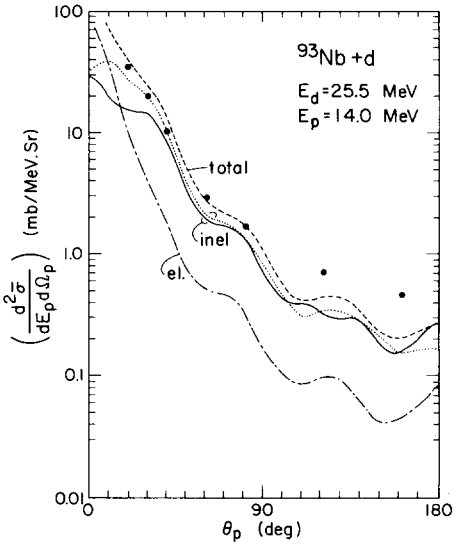

Fig. 1. The double differential cross sections ($d^2\bar{\sigma}/dE_p d\Omega_p$) for d + ^{93}Nb. The dotted and the full lines denote the inelastic breakup cross sections obtained by the SA of ref. [2] and by the present exact formalism, respectively. The dotted–dashed line is the elastic breakup cross section. The dashed line and dots represent the theoretical and experimental value of the total inclusive cross sections, respectively. All the results except the full line are taken from ref. [2].

compared with the result obtained by the present exact formalism (full line) at E_p = 14 MeV (the maximum of the spectrum). The SA overestimates the cross section in most of the angular range, but except for the forward angles, the SA seems to hold reasonably well at this maximum cross section region of the spectrum. The elastic breakup (dotted–dashed line) and the total inclusive breakup cross sections (dashed line) calculated by Pampus et al. [2] are also plotted with the experimental results [2] of the latter (dots). The SA overpredicts the experimental result, but the exact calculation added by the elastic breakup is slightly lower than the experiment. At backward angles the discrepancy between the theoretical and the experimental results is considered to be due to the compound and the precompound components as Pampus et al. [2] pointed out.

In fig. 2, the SA results (dashed line) are compared with the exact one (full line) for the angle-integrated proton energy spectra of the inelastic breakup process of ^{62}Ni(d, p). The SA overestimates the spectrum in the entire energy region. It is rather good approxi-

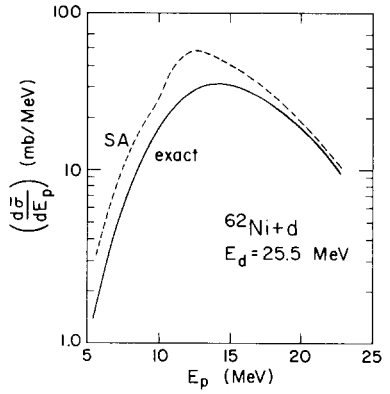

Fig. 2. The angle-integrated proton spectra of the inelastic breakup process. The full and the dashed lines denote the spectra obtained by the present method and by the SA, respectively.

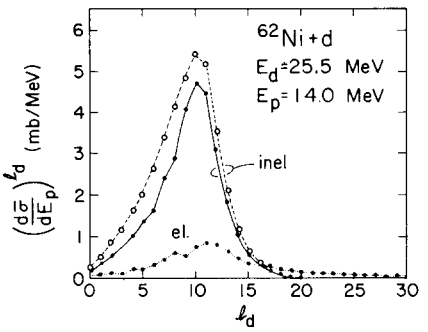

Fig. 3. The partial cross sections $(d\bar{\sigma}/dE_p)^{l_d}$. The full and the dashed lines denote the inelastic breakup cross sections calculated by the present method and by the SA, respectively. The dotted line denotes the elastic breakup cross section.

mation at large E_p, but gives much larger results (even by factor of 2) at low E_p.

We confirmed that the replacement $\int_{R_c}^{\infty} h_l$ [...] $\times r\mathrm{d}r$ in eq. (25) by $\int_0^{\infty}(\chi_l - j_l)$ [...] $\mathrm{d}r$ in the SA changes the angle integrated cross section only less than a few percent for $E_p > 12$ MeV where $R_c = 7.4$ fm is used. We compare the partial cross sections, $(\mathrm{d}\bar{\sigma}/\mathrm{d}E_p)^{l_d}_{\mathrm{inel}}$, of ^{62}Ni(d, p) in fig. 3. The sharp peak at $l_d = 10$ indicates a surface-reaction character of the inelastic-breakup reaction at the energies considered. ($E_d = 25.5$ MeV, $E_p = 14$ MeV). This is in contract to a spreading to a very wide partial wave range of the elastic-breakup partial cross sections which are also plotted (dotted line). The SA (dashed line) reproduces well the exact partial cross sections for high l_d but poor for low l_d.

We remark that for low-energy neutron (corresponding to high E_p), the imaginary part $W_n(r)$ of the neutron optical potential is surface peaked, and this is another reason for the surface reaction character of the inelastic breakup process as is seen from our formalism.

We conclude that the present DWBA breakup formalism is so easily handled exactly that one needs not rely on the surface approximation which is good at high E_p but not so good at low E_p in the present analysis.

The authors thank Dr.M. Igarashi for providing subroutines useful for Vincent and Fortune's method.

They also thank Professor T. Tamura, Professor T. Udagawa, Professor T. Terasawa and Dr. A. Ichimura for valuable discussions. Numerical calculations were performed by HITAC-M200 at the computer center, University of Tokyo. M.I. acknowledges Professor P. Siemens and Professor K. Nagatani for their hospitality during his stay at Texas A&M University.

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